GEOMETRY AND COMBINATORICS OF MATROIDS

RUBY ORTIZ

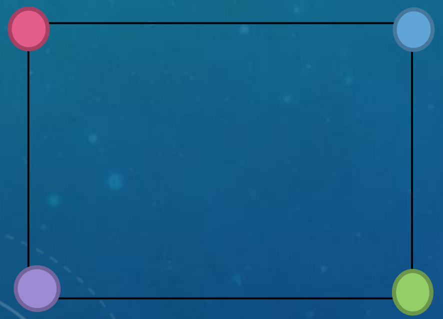
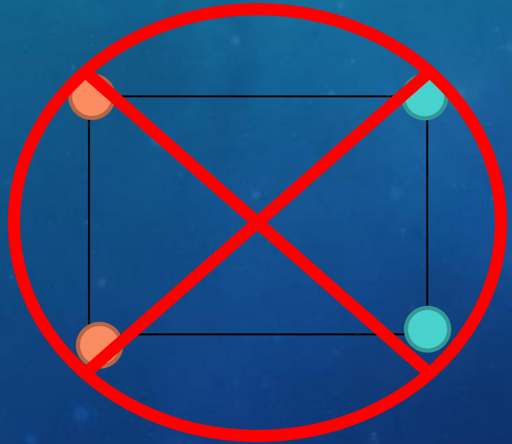
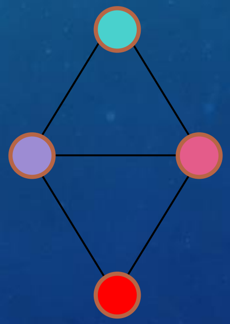
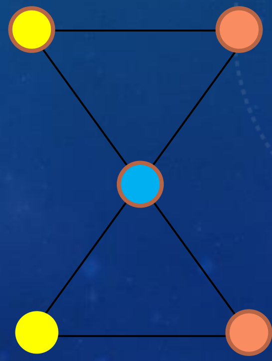
MENTOR: DR. NICOLA TARASCA

CHROMATIC POLYNOMIALS OF A GRAPH

- The Proper q-coloring
  - When no two adjacent vertices have the same color

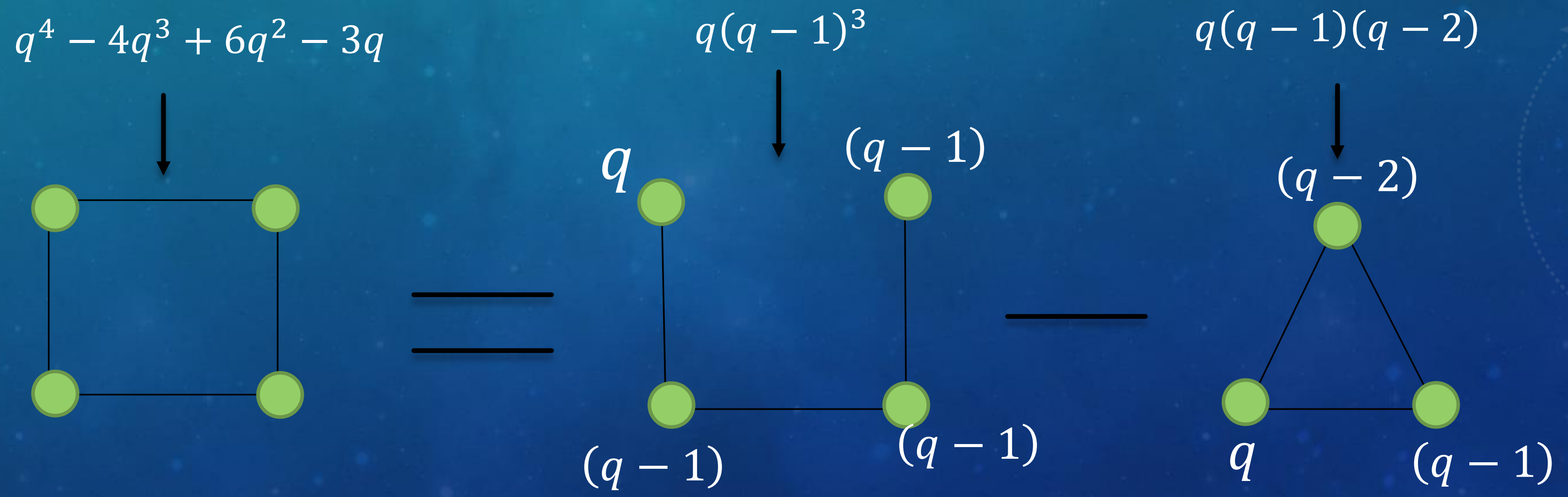
- Chromatic Polynomial
  - The number of ways a graph’s vertices can be colored so that the adjacent vertices are different colors

$$X_G(q) = the\ number\ of\ proper\ q - colorings$$
$$q = the\ number\ of\ colors$$

CHROMATIC POLYNOMIALS OF A GRAPH

- This idea stems from the Four Color Theorem
- Whitney [1932] found that the chromatic polynomial is indeed a polynomial.
  - $X_G(q)/q = a_0(G)q^d - a_1(G)q^{d-1} + \cdots + (-1)^d a_d(G)$
- Which was then computed by deletion-contraction relation by Hoggar [1974]
  - $X_G(q) = X_{G\backslash e}(q) - X_{G/e}(q)$



GOALS

- In the present, I am proving how the graph of a deleted edge's chromatic polynomial minus the graph of a contracting edge's chromatic polynomial calculates the chromatic polynomial of the graph.
- In the future, I am using this special case to understand the Theorem of Matroids and get a definition for Matroids.
- The end goal is to proof Rota's unimodality conjecture: "If $w_k(M)$ is the number of rank k flats of a rank d matroid M, then the sequence $w_0(M), \dots, w_d(M)$ is unimodal."

ACKNOWLEDGMENTS

- Thanks to MAA/NREUP program through the NSF grant DMS-1652506 for the support and funding this research.
- Thanks to Dr. Nicola Tarasca for the support in this research.
- Thanks to DIMACS and Dr. Lazaros Gallos for giving me this opportunity.

CITATIONS

- Huh, J. (2018). Combinatorial Applications Of The Hodge-Riemann Relations. Not yet published.